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TORSIONAL VIBRATION ANALYSIS OF A MODERN MARINE ENGINE SHAFT LINE USING THE MATRIX METHOD

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Abstract: Torsional oscillations remain among the most delicate structural challenges encountered during the dynamic evaluation and operational assessment of marine propulsion shafting networks. The periodic torque fluctuations originating from cylinder pressure cycles and reciprocating component inertia inherently introduce variable torsional loads capable of inducing material fatigue across crankshaft journals, intermediate shafting, and propulsion elements. This study investigates the forced torsional vibration response of a modern electronically controlled two-stroke marine engine, specifically the low-speed Wärtsilä 6X52 unit deployed across commercial ocean-going vessels like Handymax tankers and Ultramax bulk carriers. To model the continuous mechanical system, an equivalent discrete assembly comprising 8 lumped mass elements and 7 elastic shaft sections was established. The mathematical formulation integrates internal shaft hysteresis, main bearing dissipation, hydrodynamic propeller resistance, and the complex harmonic torque components linked to the dominant order $k = 6$. By implementing a multiplicative state-vector matrix scheme, the complex angular displacement field and corresponding spatial phase envelope were computed for every system node. The numerical predictions confirm the capacity of the matrix model to capture subtle dynamic phenomena, accurately pinpointing the kinematic vibration node near Cylinder 6 and yielding realistic deformation magnitudes along the entire propulsion shafting.

Key words: complex displacement, harmonic excitation, marine propulsion, matrix transfer method, torsional vibrations, Wärtsilä 6X52.

1. INTRODUCTION

The ongoing transformation of ocean transport toward enhanced energy conversion efficiency, lower fuel consumption, and compliance with rigorous international environmental mandates has accelerated fundamental changes in low-speed two-stroke marine engine architecture. Modern propulsion plants have increasingly shifted away from traditional mechanical camshaft configurations in favor of fully electronically controlled Common Rail system architectures, such as Wärtsilä Flex technology. By replacing mechanical drive chains and fuel injection pumps with electronically managed hydraulic actuators, modern engines gain exceptional flexibility over fuel injection timing and exhaust valve actuation. Beyond combustion performance, this structural shift influences running gear design through lighter, high-strength pistons and connecting rods, effectively altering the inertial properties of the reciprocating masses. Nevertheless, the shafting assembly remains continuously exposed to dynamic excitations driven by alternating cylinder gas pressures and mass inertia forces during operation.

A rigorous evaluation of torsional vibration dynamics forms a mandatory milestone during the structural design, classification approval, and operational tuning of marine shafting installations. Periodic combustion torques acting on individual crankshaft throws generate fluctuating shear stresses that propagate along the entire propulsion assembly. Should any major harmonic excitation frequency align with one of the natural torsional frequencies of the mechanical system, resonant amplification occurs. Such condition can dramatically elevate cyclic shear stress levels, leading to accelerated fatigue cracking, keyway deformation, or severe structural failure of the crankshaft and intermediate shafting.

To capture these dynamic responses, various computational tools have evolved over time, ranging from traditional iterative Holzer tabular schemes to complex multi-body finite element formulations. Within this spectrum, multiplicative matrix transfer techniques—particularly those rooted in Galati matrix formulations—offer a balanced compromise between mathematical elegance, computational efficiency, and structural accuracy for multi-mass torsional networks. By grouping discrete mass dynamics, section elasticity,

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material hysteresis, and fluid friction into complex transfer operators, the overall equilibrium of the continuous shafting line can be resolved sequentially within a structured matrix domain.

The current study employs this multiplicative matrix approach to quantify the forced torsional response of the electronically controlled Wärtsilä 6X52 propulsion engine. Through an analysis of its dynamic mass distribution, stiffness properties, and harmonic excitation components, this research maps out the exact angular displacement field, nodal position, and complex phase trajectory established along the propulsion shaft line under nominal operating conditions.

2. MATRIX METHOD PRINCIPLE

The state-vector matrix transfer method provides a compact, computationally robust framework for evaluating the torsional response of complex lumped-mass propulsion systems. By representing the continuous shaft line as a series of discrete inertial nodes connected through elastic and damping elements, this formulation integrates point transitions across individual disks and field transfer across shaft segments into unified complex operators. This approach allows for the systematic propagation of kinematic and dynamic state variables—such as complex angular displacements and internal shaft moments—along the entire shaft line while rigorously satisfying boundary conditions at the boundaries.

The general motion of the equivalent oscillatory system representing the shaft line, reduced according to [1], can be expressed in the following matrix form:

$$[J]\cdot\{\phi\}+[C]\cdot\{\phi\}+[K]\cdot\{\phi\}=\{m\} \tag{1}$$

where [J] denotes the diagonal matrix of mass moments of inertia reduced to the axis of rotation, [K] denotes the banded matrix of the torsional stiffnesses of the shaft sections between two disks, and [C] denotes the matrix of damping coefficients at the engine's main bearings. The symbols {m} and $\{\phi\}$ represent the column vectors of excitation moments and torsional angular displacements, respectively. The forms of these matrices are therefore given as follows:

$$[J]=\begin{bmatrix} J_1 & 0 & \dots & 0 \\ 0 & J_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & J_n \end{bmatrix} \tag{2}$$

$$[C]=\begin{bmatrix} C_1 & -C_1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ -C_1 & C_1+C_2 & -C_2 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -C_i & C_i+C_{i+1} & -C_{i+1} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & C_{n-1}+C_n & -C_n \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & \dots & -C_n & C_n \end{bmatrix} \tag{3}$$

$$\{\phi\}=[\phi_1 \quad \phi_2 \quad \cdots \quad \phi_n]^T \tag{4}$$

Therefore, the matrix $[K]$ has the same structure as $[C]$, and the vector $\{m\}$ has the same structure as $\{\phi\}$, with the observation that the excitation torques act only on the first discs of the system with n degrees of freedom, corresponding to the number of engine cylinders.

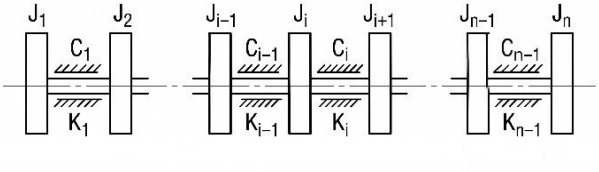


Figure 1 Equivalent lumped-mass dynamic model of the propulsion shaft line

By isolating an arbitrary i-th disc along with the subsequent massless elastic shaft segment, while also accounting for the damping in the piston-cylinder assembly as well as the left-hand torsional moment, having a complex form module:

$$\bar{M}_i=M_i^r+jM_i^i,\quad j=\sqrt{-1} \tag{5}$$

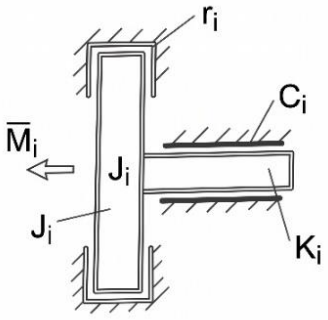


Figure 2 Lumped disk element j with external damping, torsional stiffness, and harmonic excitation

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where M_i^r and M_i^i denote the real and imaginary parts of the complex torsional moment, respectively. Based on the elements shown in Figure 2, the point matrix and the field matrix can be defined, using the notation adopted in [3], as follows:

$$[P]_i = \begin{bmatrix} \phi^r & M^r & \phi^i & M^i & 1 \\ 1 & 0 & 0 & 0 & 0 \\ -J\Omega^2 & 1 & -r\Omega & 0 & -M^r \\ 0 & 0 & 1 & 0 & 0 \\ r\Omega & 0 & -J\Omega^2 & 1 & -M^i \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i \quad (6)$$

$$[F]_i = \begin{bmatrix} \phi^r & M^r & \phi^i & M^i & 1 \\ 1 & \frac{K}{K^2+C^2\Omega^2} & 0 & \frac{C\Omega}{K^2+C^2\Omega^2} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{C\Omega}{K^2+C^2\Omega^2} & 1 & \frac{K}{K^2+C^2\Omega^2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i \quad (7)$$

By generalizing the aspects presented in [2], the following relationship is obtained:

$$[Z]_i^D = [F]_i \cdot [P]_i \cdot [Z]_{i-1}^D \quad (8)$$

where $[Z]_i^D$ denotes the right-hand state vector (superscript "D") of the i-th element shown in Figure 2 as a function of that of the preceding element, having a complex form, denoted by the overbar:

$$[\bar{Z}]_i = \{\bar{\phi} \quad \bar{M} \quad \vdots \quad 1\}^T \quad (9)$$

or the real one:

$$\left[\overline{(Z)}\right]_i = \left\{\overline{(\phi)(M)} : 1\right\}^T \quad (10)$$

where the superscripts r and i have the same meaning as before, and T denotes the matrix transposition operation.

Applying relationship (8) in complex form to the entire system shown in Figure 1, we obtain:

$$[\bar{Z}]_n^D = [\bar{P}]_n \cdot \prod_{i=1}^{n-1} [\bar{F}]_i [\bar{P}]_i \cdot [\bar{Z}]_1^S = [\bar{B}] \cdot [\bar{Z}]_1^S \quad (11)$$

The operations specified by relationship (11) are considerably reduced by applying the left boundary condition, namely that the left-hand torsional moment of the system (superscript "S") is zero:

$$\bar{M}_1^S = 0 \quad \text{or} \quad M_1^{r\,S} = M_1^{i\,S} = 0 \quad (12)$$

which leads to the elimination of the second and fourth columns from the calculations. The presented matrix method can be arranged in the following convenient form:

$$\begin{bmatrix} 1 & 0 & 0 \\ -J\Omega^2 & -r\Omega & -M^I \\ 0 & 1 & 0 \\ r\Omega & -J\Omega & -M^I \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi^I \\ \phi^i \\ 1 \end{bmatrix}_1^S = [Z]_2^D$$

$$\begin{bmatrix} 1 & \frac{K}{K^2+C^2\Omega^2} & 0 & C\Omega & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -C\Omega & 1 & \frac{K}{K^2+C^2\Omega^2} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 & -a_3 & a_5 \\ a_2 & -a_4 & a_6 \\ a_3 & a_2 & a_7 \\ a_4 & a_2 & a_8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi^x \\ \phi^i \\ 1 \end{bmatrix}_1^S = [\hat{z}]_2^S$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -J\Omega^2 & 1 & -I\Omega & 0 & -M^T \\ 0 & 0 & 1 & 0 & 0 \\ I\Omega & 0 & -\Omega\Omega^2 & 1 & -M^1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_{i_2} & -A_{i_2} & A_{i_s} \\ A_{i_2} & -A_{i_4} & A_{i_s} \\ A_{i_3} & A_{i_3} & A_{i_7} \\ A_{i_4} & A_{i_2} & A_8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi^{\hat{I}} \\ \phi^i \\ 1 \end{bmatrix} = [\hat{Z}]_1^D$$

$$\begin{bmatrix} 1 & \frac{K}{K^2+C^2\Omega^2} & 0 & C\Omega & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -C\Omega & 1 & \frac{K}{K^2+C^2\Omega^2} & 0 \\ 0 & 0 & 0 & \frac{1}{0} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_{i_1} & -B_{i_2} & B_{i_s} \\ B_{i_2} & -B_{i_4} & B_{i_s} \\ B_{i_1} & B_{i_2} & B_{i_7} \\ B_{i_4} & B_{i_2} & B_{i_6} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ \phi^T \\ \phi^I \\ 1 \end{bmatrix} = [\hat{Z}]_{i+1}^S$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -J\Omega^2 & 1 & -I\Omega & 0 & -M^s \\ 0 & 0 & 1 & 0 & 0 \\ r\Omega & 0 & -J\Omega^2 & 1 & -M^j \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 & -b_3 & b_5 \\ b_2 & -b_4 & b_6 \\ b_3 & b_1 & b_7 \\ b_4 & b_2 & b_8 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \phi^x \\ \phi^1 \\ 1 \end{bmatrix}_1^S = [\hat{z}]_{n_8} \quad (13)$$

In (13), the coefficients a_j, $j = \overline{1,8}$ are determined using the following relationships:

$$a_1; \quad a_2 = -J\Omega; \quad a_3 = 0; \quad a_4 = r\Omega; \quad a_5 = C\frac{\Omega}{K^2 + C^2\Omega^2}$$

$$a_6 = -\frac{J\Omega^3 C}{K^2 + C^2\Omega^2}; \quad a_7 = \frac{K}{K^2 + C^2\Omega^2}; \quad a_8 = 1 - \frac{(JK - rC)\Omega^2}{K^2 + C^2\Omega^2} \quad (14)$$

where A_{i_j}, B_{i_j}, $i = \overline{2, n-1}$, $j = \overline{1,8}$ are determined successively through the iterative procedure presented in (13). We specify that in the previous relationships, Ω denotes the angular frequency of the exciting moments. Imposing now the right boundary conditions, analogous to (12):

$$\bar{M}_n^D = 0 \quad \text{or} \quad M_n^{r\,D} = M_n^{i\,D} = 0 \quad (15)$$

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which leads to the system of equations derived from (13):

$$b_2\phi_1^r - b_4\phi_1^i + b_6 = 0 \quad (16)$$
$$b_4\phi_1^r + b_2\phi_1^i + b_8 = 0 \quad (17)$$

from which the components ϕ_1^r, ϕ_1^i of the state vector $[\bar{Z}_1]^S$ are obtained. Step by step, the components of the complex state vectors of the entire system will be determined, and consequently the shape of the deformed elastic line under damped torsional vibrations.

3. NUMERICAL APPLICATION AND CASE STUDY: THE WÄRTSILÄ 6X52 ELECTRONICALLY CONTROLLED MARINE ENGINE

For the numerical application and validation of the developed state-vector matrix framework, the propulsion shafting assembly driven by a modern Wärtsilä 6X52 marine engine was selected as the reference installation. The Wärtsilä 6X52 is an electronically controlled, two-stroke, low-speed marine diesel engine designed to serve as the main propulsion unit across a broad range of ocean-going merchant vessels, including Handymax tankers, Ultramax bulk carriers, and feeder container ships.

Modern marine propulsion architectures operate under increasingly demanding operational profiles dictated by global decarbonization targets and strict energy efficiency frameworks, such as the IMO Energy Efficiency Existing Ship Index (EEXI) and Carbon Intensity Indicator (CII). To meet these mandates, the engine design relies on a camshaft less Common Rail system incorporating electronic control units (ECUs) to manage high-pressure fuel injection and exhaust valve actuation. This structural configuration provides exceptional flexibility in adjusting combustion mapping, cylinder firing strategies, and operational tuning relative to varying ship speeds and propeller loads.

From a dynamic standpoint, the elimination of heavy mechanical timing trains, drive chains, and camshaft assemblies drastically simplifies the overall system geometry while altering the mass distribution at the engine forward end. Furthermore, modern running gear components—including pistons, crossheads, and connecting rods—are engineered using high-strength alloy materials, yielding optimized reciprocating mass profiles without compromising mechanical stiffness.

However, despite these modern structural and electronic refinements, the fluctuating gas forces within the combustion chambers, combined with the primary and secondary inertial forces of the running gear, continue to generate substantial periodic excitation torques. These dynamic moments propagate along the multi-mass mechanical system, inducing complex coupled torsional oscillations across the crankshaft, intermediate shafting, and propulsion unit. Consequently, evaluating the forced torsional response of the Wärtsilä 6X52 shafting line using the multiplicative matrix transfer method provides critical analytical insights into the angular displacement field, spatial phase envelopes, and exact location of the kinematic vibration nodes under nominal service conditions.

Input data for the equivalent torsional system:

The continuous mechanical system of the shaft line was discretized into an equivalent model consisting of $n = 8$ lumped masses and 7 elastic shaft sections. The dynamic calculation parameters for the **Wärtsilä 6X52** equivalent system are summarized as follows:

- Lumped Moments of Inertia (J_i):
  - Cylinders 1 through 6: $J_1 = J_2 = J_3 = J_4 = J_5 = J_6 = 8450.00\ \text{N} \cdot \text{m} \cdot \text{s}^2$
  - Engine Flywheel: $J_7 = 3850.00\ \text{N} \cdot \text{m} \cdot \text{s}^2$
  - Propeller (including entrained water mass): $J_8 = 71706.224\ \text{N} \cdot \text{m} \cdot \text{s}^2$
- Torsional Stiffnesses ($K_{i,i+1}$):
  - Crankshaft sections between cylinders: $K_1 = K_2 = K_3 = K_4 = K_5 = 0.105 \times 10^9\ \text{N} \cdot \text{m/rad}$
  - Cylinder 6 to Flywheel section: $K_6 = 0.585 \times 10^9\ \text{N} \cdot \text{m/rad}$
  - Intermediate and Propeller shaft line: $K_7 = 0.128 \times 10^9\ \text{N} \cdot \text{m/rad}$
- Damping Coefficients **($r_i, C_{i,i+1}$):**
  - External damping at disk nodes: $r_1 = \cdots = r_6 = 3003.5373\ \text{N} \cdot \text{m} \cdot \text{s}$, $r_7 = 432.91\ \text{N} \cdot \text{m} \cdot \text{s}$, $r_8 = 243909.84\ \text{N} \cdot \text{m} \cdot \text{s}$
  - Internal shaft section damping: $C_1 = \cdots = C_7 = 19823.34\ \text{N} \cdot \text{m} \cdot \text{s}$
- Operating Regime and Excitation Parameters:
  - Excitation angular frequency for the major 6th order harmonic ($k = 6$) at nominal speed: $\Omega = 38.32743\ \text{s}^{-1}$.
  - Harmonic excitation moments per cylinder, derived from the cylinder indicator diagram and crank star arrangement (accounting for the firing order phase shifts of a 6-cylinder two-stroke engine):
  - $$M_1^r = M_2^r = M_3^r = -M_4^r = -M_5^r = -M_6^r = 145200.00\ \text{N} \cdot \text{m}$$
  - $$M_1^i = M_2^i = M_3^i = -M_4^i = -M_5^i = -M_6^i = 210800.00\ \text{N} \cdot \text{m}$$

Applying the multiplicative set of operations (13) through the developed matrix calculation algorithm yields the following numerical results for the Wärtsilä 6X52 engine system:

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Table 1 Complex Angular Displacements (ϕ_j^r, ϕ_j^i) across the Shaft Line Nodes

Disk No. j	ϕ_j^r [rad]	ϕ_j^i [rad]
1	0.0396576	0.0000000
2	0.0373680	0.0003602
3	0.0330602	0.0010425
4	0.0272446	0.0019746
5	0.0158144	0.0021826
6	0.0001209	0.0016641
7	-0.0031295	0.0014764
8	-0.0178419	0.0004859

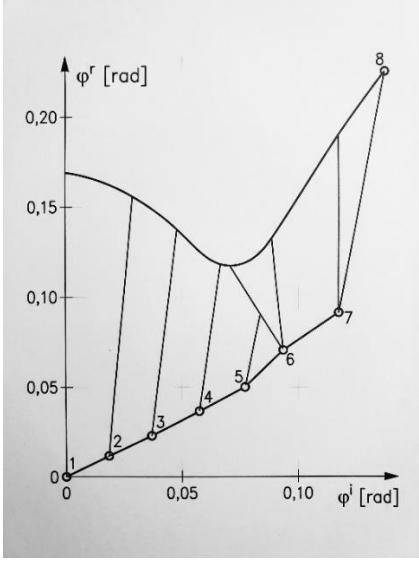


Figure 3 Complex phase trajectory and amplitude envelope of the torsional angular displacements along the shaft line

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4. CONCLUSIONS

The dynamic investigation conducted in this work demonstrates the structural reliability and mathematical simplicity of the multiplicative matrix transfer method when applied to the forced torsional response of modern marine propulsion systems. Modeling the Wärtsilä 6X52 engine shaft line as an equivalent 8-mass, 7-section discrete dynamic network enabled a continuous spatial tracking of state vectors across the entire mechanical domain, spanning from the crankshaft's free end to the propeller disc.

By formulating complex point and field transfer matrices that explicitly incorporate internal material hysteresis alongside external main bearing friction and hydrodynamic propeller resistance, the calculation framework effectively captured the dynamic behavior of the assembly under nominal service conditions. The boundary conditions imposed at the propeller node ($M_8^D = 0$) were satisfied with high numerical precision, confirming the mathematical consistency of the global transfer operator.

The computed displacement field reveals that the peak real component of the complex angular displacement occurs at Cylinder 1 ($\phi_1^r = 0.0396576$ rad), gradually attenuating along the crankshaft axis toward the aft part of the engine. A sign reversal in the real component between Cylinder 6 ($\phi_6^r = 0.0001209$ rad) and the engine flywheel ($\phi_7^r = -0.0031295$ rad) pinpoints the location of the primary kinematic node in the immediate proximity of Cylinder 6, coinciding with the global minimum of the total vibration envelope ($|\phi_6| = 0.0016685$ rad). Beyond this nodal transition, the flywheel and propeller oscillate in phase opposition relative to the forward engine cylinders, with the total displacement amplitude regaining magnitude up to $|\phi_8| = 0.0178485$ rad at the propeller. Simultaneously, the steady progression of the imaginary component ϕ_j^i along the cylinder sequence—peaking at $+0.0021826$ rad near Cylinder 5—accurately quantifies the spatial phase lag induced by energy dissipation in the main bearing lubricant films.

Overall, the matrix methodology proven here offers a practical, computationally efficient analytical tool for preliminary shafting dimensioning, torsional vibration troubleshooting, and digital monitoring of modern electronically controlled marine engines.

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6. CREdit authors statement

Conceptualization: T.A.R.

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Formal analysis: V.B.M

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Methodology: T.A.R

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